Calculus is a language. It is the language of science, and it is increasingly the language of the social sciences. Calculus is more than a language, of course. It is a vast body of knowledge, an armory of powerful analytical tools. It is a masterpiece of the human mind. Calculus occupies a special place in our world because of the precise and economical way it can describe everything from orbiting satellites and falling prices to decaying atoms and spreading diseases.

Calculus: The Language of Change guides students through this body of knowledge by teaching calculus as a language. Approached as such, calculus is immediately meaningful to students, and the meaning stays with them even when formulas fade from memory. The other aspects of calculus—the geometry, algorithms, formulas, and analysis—take on greater significance when calculus is first seen as saying something concrete about the world.

The role of calculus as a language is central to this book. We start by introducing the vocabulary of calculus. From there we move quickly to present the calculus pictorially, and then computationally, algebraically and theoretically. In this way, students begin by attaining a working facility with the language, and then go on to become comfortable with its concepts, proficient with its tools and, finally, educated in its foundations. That's the way humans master languages from infancy, and we find that a compelling model for teaching calculus.

FEATURES

- **The meaning of calculus comes first.** A language is best learned when it can be used to make meaningful statements, thus the first chapter teaches students how to use the words and symbols for the derivative and the integral. After reading just a few pages the student is using derivatives and integrals to "translate" from English to Calculus and from Calculus to English.
- **Derivatives and integrals are developed side by side throughout the text.** They are parts of a whole and thus are easier to understand when studied together. We've found that students have no diffculty working with derivatives and integrals side by side from the start, and at the end of a semester the profound relationship between them is both real and memorable.
- **Theory is offered at several levels.** Derivatives and integrals are developed informally in Chapters 1 through 4. In Chapter 5 they're given a formal but friendly treatment; this chapter can be covered in its entirety, in part, or can be omitted altogether. For the most ambitious students, the complete theory is given in Chapter 12.
- **Computing devices are incorporated generically.** We take advantage of the insights offered by graphing calculators and computer programs. Students and instructors are free to use

the devices and programs with which they're most familiar. Those who don't have their own can find all the software tools they need at the book's website, http://math.jbpub.com/calculus.

- **Examples from many disciplines offer a choice of course emphasis.** There are projects involving applications of calculus throughout the book. Additional applications can be found at the book's website. By choosing which topics to include, instructors can design a variety of courses to suit different audiences.
- **Calculus: The Language of Change is both accessible and deep.** Chapters 1 through 4 are especially easy for students to read and understand. Each of these chapters calls on a small set of precalculus skills that can be reviewed as the chapter is covered rather than in advance. At the same time, ***Calculus: The Language of Change*** moves on to sophisticated applications (pharmacokinetics, cosmology) and advanced mathematical techniques and theorems (constrained optimization, Bolzano-Weierstrass).
- **Calculus: The Language of Change is both integrated and diverse.** Each chapter of the book connects in multiple ways to previous chapters. In particular, Chapters 9, 10, and 11 weave together the ideas presented earlier in the book. At the same time, ***Calculus: The Language of Change*** moves broadly across disciplines, devoting entire sections to topics in archaeology, computer science, international relations, public policy, linguistics, and ecology.

USING CALCULUS: THE LANGUAGE OF CHANGE

Each chapter of ***Calculus: The Language of Change*** comprises a number of core sections followed by a group of optional sections. By choosing the chapters and sections to cover, an instructor can fashion a great variety of courses (see the suggested syllabi on page xiv):

- a standard, one-size-fits-all, two-semester sequence
- a high-powered course for engineers and physicists
- one or two semesters for pre-health students
- a two-semester sequence for economics and business students
- one or two semesters for liberal arts students
- a leisurely three-semester sequence
- a combined, two-semester, precalculus/calculus sequence
- an accelerated course
- a high school AB course
- a high school BC course

The approach of ***Calculus: The Language of Change*** necessitates changing the order of some topics, but only within semesters. Topics traditionally found in a Calculus I course can be found in Chapters 1–6. Those typically found in a Calculus II course are in Chapters 7–9.

Because there are differing views among calculus instructors about what to cover in Calculus II, we offer a range of choices. In addition to the topics typically found in Calculus II courses, we also include Chapter 10, a light introduction to calculus in $\mathbb{R}^3$, which ties in with some of the work in differential equations. In Chapter 11 we offer an introduction to calculus with complex numbers. This is a nice way to unify the material of the second semester. Finally, Chapter 12 provides a discussion of the deeper theories of derivatives and integrals. It can be presented on its own or as an extension of the discussion begun in Chapter 5.

As mentioned earlier, students should not need a review of precalculus material before beginning. Instead, brief reviews can accompany each of the first four chapters. A lab manual for this purpose is available from Jones and Bartlett.

TECHNOLOGY SUPPLEMENTS

Technology is highlighted throughout ***Calculus: The Language of Change***, but the book is not tied to any particular software or hardware. Most graphing calculators can be used with the book, as well as more powerful computer programs such as Mathematica, Maple, and Derive. Jones and Bartlett offers lab manuals that follow the text for these programs.

For teachers who prefer that students use common software, there are utilities available at no cost at the book's website. Students will find these utilities both easy to learn and to use. All problems in the book requiring computers can be done with our utilities, with the exception of those involving spreadsheets. The available utilities include:

- **Graph:** *Graph* is a simple graphing utility. It graphs up to five functions simultaneously.
- **Slinky:** *Slinky* solves and graphs the solutions to initial value problems. Given a set of differential equations with up to 10 variables, together with initial conditions, it will draw the solutions curves. If the system has only one dependent variable, *Slinky* offers the slope field.
- **SlinkyPlus:** *SlinkyPlus*, like *Slinky*, solves and graphs the solutions to initial value problems. It can also show solutions for higher order systems of equations. If the system has exactly two dependent variables, *SlinkyPlus* offers the phase plane.
- **GraphPlus:** *GraphPlus* is a three-dimensional graphing utility. It graphs functions of two variables over rectangular domains in five ways: as a wire frame, as a three-dimensional wire frame (3D glasses required), as a contour map, as a density plot, and as a reflecting surface. The reflecting surface (called dunes) is particularly interesting. It is unlike any program we know in that it shows not just a small portion of the surface, but all that the eye can see.

USING TECHNOLOGY WITH THE EXERCISES AND PROBLEMS

Five icons are used to denote specific types of problems within the exercise sets:

Problems with this icon require a **graphing utility**.

Problems with this icon require a **differential equations** solver.

Problems with this icon require (or lend themselves to) a **spreadsheet** such as Microsoft Excel.

Problems with this icon can be solved using **special software** available at the book's website.

Problems with this icon require a **surface graphing utility**.

The optional laboratory sections and the More Challenging Problems lend themselves well to group work. We have found that, as with learning any language, it is helpful to practice speaking it with others. We offer plenty of opportunity to do that. Of course, all of the problems—especially the general exercises—work well as individual assignments.

PRINT AND ELECTRONIC SUPPLEMENTS

A complete set of Instructor and Student supplements is available with ***Calculus: The Language of Change***, including:

- An Instructor's Manual, containing complete solutions to all the problems, notes on the material, sample test problems, and notes on precalculus review material for all of the chapters;
- Lab manuals for Mathematica and MATLAB, which closely follow the text and teach students how to integrate Computer Algebra Systems into their study of calculus;
- A Student Review Guide for Precalculus, which provides comprehensive coverage and review of the prerequisite algebra and precalculus material;
- A companion website (http://math.jbpub.com/calculus) for students featuring additional applications, exercises, and problems, as well as review material for algebra and trigonometry.

SUGGESTED SYLLABI

The following are suggestions for choosing sections to fit various audiences.

A Standard, One-size-fits-all Course

Calculus I
The core sections of Chapters 1, 2, 3, 4, and 6, with the first three sections of Chapter 5. Selected optional sections can be included to meet the needs of a particular class.

Calculus II
The core sections of Chapters 7, 8, 9, 10, and 11, with selected optional sections.

A High-powered Course for Engineers and Physicists

Calculus I
The core sections of Chapters 1, 2, 3, 4, and 6, together with some of the optional sections, such as 4.11, 4.12, 4.15, 4.16, 4.17, 6.10, and 6.12.

Calculus II
The core sections of Chapters 7, 8, 9, 10, and 11, together with some of the optional sections, such as 7.10, 7.12, 7.13, 7.14, 8.6, 9.13, 10.7, 11.5, 11.6, and 11.7.

One or Two Semesters for Pre-health Students

Calculus I

The core sections of Chapters 1, 2, 3, 4, and 6, together with optional Sections 2.6 and 3.8.

Calculus II

The core sections of Chapters 7, 8, and 9, together with some of the optional sections (such as 7.10, 7.11, 8.7, 8.9, and 9.14), and the Web application *More Pharmacokinetics*.

A Two-semester Sequence for Economics and Business Students

Calculus I

The core sections of Chapters 1, 2, 3, 4, and 6, together with some of the optional sections (such as 2.5, 3.7, 4.14, and 6.13), and the Web applications *Rich and Poor, The Wealth of Families,* and *Fuel*.

Calculus II

The core sections of Chapters 7, 8, 9, 10, and 11, together with some of the optional sections (such as 7.10, 8.7, 8.8, 9.14, 10.8, 10.10, and 11.8), and the Web applications *Measuring the Inequality of Wealth* and *Soaking the Rich*.

One or Two Semesters for Liberal Arts Students

Calculus I

The core sections of Chapters 1, 2, 3, 4, and 6, with the first three sections of Chapter 5. Include a variety of optional sections emphasizing the breadth of applications.

Calculus II

The core sections of Chapters 7, 8, 9, 10, and 11, with additional optional sections.

A Leisurely Three-semester Sequence

For schools that cover calculus in three semesters.

Calculus I

The core sections of Chapters 1, 2, 3, and 4, with most of the optional sections.

Calculus II

The core sections of Chapters 5, 6, and 7, with most of the optional sections.

Calculus III The core sections of Chapters 8, 9, 10, and 11, with additional optional sections.

A Combined, Two-semester, Precalculus/Calculus Sequence

First semester

The core sections of Chapters 1, 2, 3, with most of the optional sections. Incorporate additional applications from the Web and a review of the following areas of precalculus:

- The language of algebra, translating between English and equations
- Graphing functions and analytic geometry
- Solving equations

Second semester

The core sections of Chapters 4 and 6, with most of the optional sections. Incorporate additional applications from the Web, and a review of the following areas of precalculus:

- trigonometry
- the exponential and logarithm functions

An Accelerated Course

At Smith we have a one-semester course for students who have just completed high school AB calculus. This course covers the BC material, plus differential equations and modeling. The students are generally strong algebraically, but often have a weak idea of what calculus is. For this course, we use:

- a selection of the optional sections from the first four chapters
- Chapter 5
- The core sections of Chapters 7, 8, 9, 10, and 11, together with a few of the optional sections.

A High School AB Course

For this course we suggest the core sections of Chapters 1 through 7, together with an assortment of optional sections.

A High School BC Course

For this course we suggest the core sections of Chapters 1 through 9, together with an assortment of optional sections.

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James Henle