PROJECT FOR SECTION 2.7

Potassium-Argon Dating

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The mineral potassium, whose chemical symbol is K, is the eighth most abundant element in the earth's crust, making up about 2% of it by weight, and one of its naturally occurring isotopes, K-40, is radioactive. The radioactive decay of K-40 is more complex than that of carbon-14 because each of its atoms decays through one of two different nuclear decay reactions into one of two different substances: the mineral calcium-40 (Ca-40) or the gas argon-40 (Ar-40). Dating methods have been developed using both of these decay products. In each case, the age of a sample is calculated using the ratio of two numbers: the amount of the **parent** isotope K-40 in the sample and the amount of the **daughter** isotope (Ca-40 or Ar-40) in the sample that is **radiogenic**, that is, that originated from decay of the parent isotope after the formation of the rock.

The amount of K-40 in a sample is easy to calculate. K-40 comprises 1.17% of naturally occurring potassium, and this small percentage is distributed quite uniformly, so that the mass of K-40 in the sample is just 1.17% of the total mass of potassium in the sample, which can be measured. But for several reasons it is complicated, and sometimes problematic, to determine how much of the Ca-40 in a sample is radiogenic. In contrast, when an igneous rock is formed by volcanic activity, all of the argon (and other) gas previously trapped in the rock is driven away by the intense heat. At the moment when the rock cools and solidifies, the gas trapped inside the rock has the same composition as the atmosphere. There are three stable isotopes of argon, and in the atmosphere they occur in the following relative abundances: 0.063% Ar-38, 0.337% Ar-36, and 99.60% Ar-40. Of these just one, Ar-36, is not created radiogenically by the decay of any element, so any Ar-40 in excess of 99.60/.337 = 295.5 times the amount of Ar-36 must be radiogenic. So the amount of radiogenic Ar-40 in the sample can be determined from the amounts of Ar-40 and Ar-36 in the sample, which can be measured.

Assuming that we have a sample of rock for which the amount of K-40 and the amount of radiogenic Ar-40 have been determined, how can we calculate the age of the rock? Let $P(t)$ be the amount of K-40, $A(t)$ the amount of radiogenic Ar-40, and $C(t)$ the amount of radiogenic Ca-40 in the sample as functions of time t in years since formation of the rock. Then the decay of K-40 is modelled by the differential equations

$$\frac{dA}{dt} = k_A P(t)$$

$$\frac{dC}{dt} = k_C P(t)$$

$$\frac{dP}{dt} = -(k_A + k_C)P(t)$$

where $k_A = 0.581 \times 10^{-10}\ \text{yr}^{-1}$ and $k_C = 4.962 \times 10^{-10}\ \text{yr}^{-1}$.

1. Find a formula for $P(t)$. What is the half-life of K-40?
2. Show that

$$A(t) = \frac{k_A}{k_A + k_C} P(t)\left(e^{(k_A + k_C)t} - 1\right).$$

3. After a very long time (i.e., let $t \to \infty$), what percentage of the K-40 originally present in the sample decays to Ar-40 and what percentage decays to Ca-40?
4. Show that the age t of the rock as a function of the present amounts $P(t)$ of K-40 and $A(t)$ of radiogenic Ar-40 in the sample is

$$t = \frac{1}{k_A + k_C} \ln\left[\frac{A(t)}{P(t)}\left(\frac{k_A + k_C}{k_A}\right) + 1\right].$$

5. Suppose it is found that each gram of a rock sample contains 8.6×10^{-7} grams of radiogenic Ar-40 and 5.3×10^{-6} grams of K-40. How old is the rock?