▸▸▸ PROJECT FOR SECTION 8.15

Temperature Dependence of Resistivity

Anton M. Jopko, Ph.D.
Department of Physics and Astronomy, McMaster University

A conductor of length L and uniform cross-sectional area A has a resistance R given by $R = \rho L/A$, where the conductor is made of a material with resistivity ρ. The resistivity is not constant for all temperatures of the conductor, however. When current flows through the conductor, heat is generated, which raises the temperature of the conductor. This process is called Joule heating. In general, the higher the temperature, the higher the resistivity and ultimately the resistance. This means resistivity must be known at the working temperature of the conductor. We model the resistivity at temperature T_c of the conductor by a quadratic function given by

$$\rho(T_c) = \rho_0 + \alpha(T_c - T_0) + \beta(T_c - T_0)^2$$

where T_c is the temperature of the conductor in degrees Celsius, T_0 is the room temperature, and ρ_0 is the resistivity at room temperature. The coefficients ρ_0, α, and β are to be determined by experiment.

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Tungsten is a conductor with a very high melting point that is used to make filaments in incandescent light bulbs. Suppose the data in the table are measured for the resistivity of tungsten. In the problems that follow, we describe a **least squares procedure** to find the values of ρ_0, α, and β. We will assume that $T_0 = 20^\circ$C.

T_c (°C)	Resistivity (Ω-m) × 10^{-8}
20	5.60
40	5.65
80	5.70
200	7.82
500	11.1
700	20.2
1000	30.5

Related Problems

We wish to fit data points (x_i, y_i) by using the general quadratic equation $y = ax^2 + bx + c$ in the least squares sense. With only three data points, there would be no need for the least squares procedure. In our case, we have seven data points.

1. Construct the column vector $\mathbf{Y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_7 \end{pmatrix}$ and the matrix

$$\mathbf{A} = \begin{pmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ \vdots & \vdots & \vdots \\ x_7^2 & x_7 & 1 \end{pmatrix}.$$

2. Let the column vector $\mathbf{X}^* = \begin{pmatrix} a^* \\ b^* \\ c^* \end{pmatrix}$ contain the least square coefficients. Calculate the vector

$$\mathbf{X}^* = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{Y}.$$

3. Using the least squares quadratic equation, predict the resistivity of tungsten at 300°C.
4. If a tungsten conductor at room temperature has a resistance of 5 ohms, use your result from Problem 3 to predict its resistance at a temperature of 300°C.
5. Find the RMS error of the least squares quadratic,

$$\sqrt{\frac{1}{n}\sum_{i=1}^{n}(\mathbf{Y}_i - \mathbf{Y}_i^*)^2},$$

where $\mathbf{Y}^* = \mathbf{A}\mathbf{X}^*$ is the least squares value of $\mathbf{Y}$.
6. Explain, in rough terms, what the RMS error means.
7. Predict the resistivity of the tungsten conductor at 2000°C. How reliable is this value?