▸▸▸ PROJECT FOR SECTION 3.10

The Ballistic Pendulum

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Historically, to maintain quality control over munitions (bullets) produced by an assembly line, the manufacturer would use a **ballistic pendulum** to determine the muzzle velocity of a gun—that is, the speed of a bullet as it leaves the barrel. The ballistic pendulum (invented in 1742) is simply a plane pendulum consisting of a rod of negligible mass to which a block of wood of mass m_w is attached. The system is set in motion by the impact of a bullet, which is moving horizontally at the unknown velocity v_b; at the time of the impact, $t = 0$, the combined mass is $m_w + m_b$, where m_b is the mass of the bullet embedded in the wood. In Section 3.10, we saw that in the case of small oscillations, the angular displacement $\theta(t)$ of a plane pendulum shown in Figure 1 is given by the linear differential equation $\theta'' + (g/l)\theta = 0$, where $\theta > 0$ corresponds to motion to the right of vertical. The velocity v_b can be found by measuring the height h of the mass $m_w + m_b$ at the maximum displacement angle θ_{max} shown in Figure 1.

Intuitively, the horizontal velocity V of the combined mass (wood plus bullet) after impact is only a fraction of the velocity v_b of the bullet: $V = \left(\dfrac{m_b}{m_w + m_b}\right)v_b$. Now recall that a distance s traveled by a particle moving along a circular path is related to the radius l and central angle θ by the formula $s = l\theta$. By differentiating the last formula with respect to time t, it follows that the angular velocity ω of the mass and its linear velocity v are related by $v = l\omega$. Thus the initial angular velocity ω_0 at the time t at which the bullet impacts the wood block is related to V by $V = l\omega_0$ or $\omega_0 = \left(\dfrac{m_b}{m_w + m_b}\right)\dfrac{v_b}{l}$.

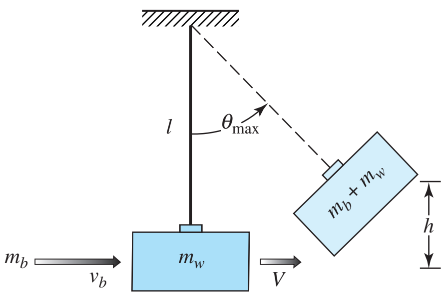

Figure 1 Ballistic pendulum

Related Problems

1. Solve the initial-value problem

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\theta = 0, \quad \theta(0) = 0, \quad \theta'(0) = \omega_0.$$

2. Use the result from Problem 1 to show that

$$v_b = \left(\frac{m_w + m_b}{m_b}\right)\sqrt{lg}\;\theta_{max}.$$

3. Use Figure 1 to express $\cos\theta_{max}$ in terms of l and h. Then use the first two terms of the Maclaurin series for $\cos\theta$ to express θ_{max} in terms of l and h. Finally, show that v_b is given (approximately) by

$$v_b = \left(\frac{m_w + m_b}{m_b}\right)\sqrt{2gh}.$$

4. Use the result from Problem 3 to find v_b when $m_b = 5$ g, $m_w = 1$ kg, and $h = 6$ cm.