Application Essay

Forebody Drag of Bluff Bodies

Robert E. Breidenthal
University of Washington

The shape of the front end of automobiles and especially trucks has markedly changed over the past two decades. Prior to this change, cars and trucks were boxy at the front, with sharp corners and prominent radiators. See [1] for vehicle aerodynamics and its history and [2] for the general subject of aerodynamic drag from a former Messerschmitt designer. However, modern vehicles are now much different, with rounded corners at the front. This has dramatically lowered the aerodynamic drag of these vehicles at highway speeds, with a corresponding increase in cruise fuel economy. In this example, taken from class notes of Professor Anatol Roshko at Caltech, we will explore the underlying physics of the flow around the front of these vehicles.

For simplicity, we consider first the flow of a frictionless fluid at speed U past a flat-sided bluff body. The flowfield is readily calculated using the **Schwarz–Christoffel transformation.** A forward-facing step is illustrated in Figure 1, which represents the upper half of a blunt-faced bluff body. Using the Schwarz–Christoffel transformation, the potential flow past the step is calculated.

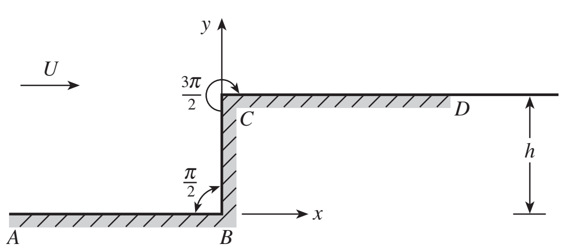

Figure 1 z-plane

First, the actual flow in the physical z-plane is transformed to the ζ-plane (where the speed is U′) by the transformation

$$\frac{dz}{d\zeta} = k\left(\frac{\zeta-1}{\zeta+1}\right)^{1/2} = k\,\frac{\zeta-1}{\sqrt{\zeta^2-1}} = k\left(\frac{\zeta}{\sqrt{\zeta^2-1}} - \frac{1}{\sqrt{\zeta^2-1}}\right).$$

See Figure 2.

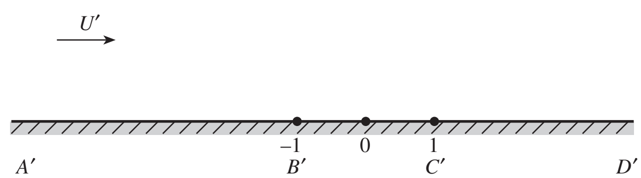

Figure 2 ζ-plane

Integrating, we find

$$z = k\left[\sqrt{\zeta^2-1} - \ln(\zeta + \sqrt{\zeta^2-1})\right] + \text{const.}$$

Evaluating the constants, at $z = 0$, $\zeta = -1$, so const. $= ki\pi$. At $z = ih$, $\zeta = 1$, so that $k = h/\pi$. Therefore

$$z = \frac{h}{\pi}\left[\sqrt{\zeta^2-1} - \ln(\zeta + \sqrt{\zeta^2-1})\right] + ih.$$

Now, the complex velocity is

$$w(z) = u - iv = \frac{dF}{dz} = \frac{dF}{d\zeta}\,\frac{d\zeta}{dz} = \frac{U'}{k\left(\dfrac{\zeta-1}{\zeta+1}\right)^{1/2}},$$

where the velocity is U' in the ζ-plane. Thus

$$w(z) = \left(\frac{\zeta+1}{\zeta-1}\right)^{1/2} U.$$

For ζ real going to infinity, z goes to infinity, and w goes to U. For ζ real going to minus infinity, z goes to minus infinity, and w goes to U. We conclude that $U = U'/k = \pi U'/h$. However, we require $U = U'$, so $h = \pi$.

The pressure distribution on the boundary is then determined from Bernoulli's equation. The pressure coefficient c_p is a dimensionless measure of the pressure,

$$c_p = \frac{p - p_\infty}{\tfrac{1}{2}\rho U^2} = 1 - \frac{\text{vel}^2}{U^2},$$

where the magnitude of the velocity is defined by

$$\text{vel}^2 = u^2 + v^2,$$

and the fluid density is ρ. In Figure 1, on the segments AB and CD, the vertical velocity v is zero, so

$$c_p = 1 - \frac{u^2}{U^2}.$$

On BC, the horizontal velocity component $u = 0$, so

$$c_p = 1 - \frac{v^2}{U^2}.$$

The total drag force is the integral of the pressure on the face of the step

$$D = \tfrac{1}{2}\rho U^2 \int_0^h c_p\, dy.$$

On BC, $\zeta = \xi$, where $-1 < \xi < 1$. The complex velocity

$$u - iv = U\left(\frac{\xi+1}{\xi-1}\right)^{1/2} = iU\left(\frac{1+\xi}{1-\xi}\right)^{1/2}.$$

Also on BC, $u = 0$ and

$$v = U\left(\frac{1+\xi}{1-\xi}\right)^{1/2}.$$

Consequently,

$$c_p = 1 - \frac{1+\xi}{1-\xi}.$$

From the transformation earlier,

$$\frac{dz}{d\zeta} = i\frac{dy}{d\xi} = k\left(\frac{\xi-1}{\xi+1}\right)^{1/2} = ik\left(\frac{1-\xi}{1+\xi}\right)^{1/2},$$

so that

$$\frac{dy}{d\xi} = k\left(\frac{1-\xi}{1+\xi}\right)^{1/2} = \left(\frac{1-\xi}{1+\xi}\right)^{1/2}.$$

The drag becomes

$$D = \tfrac{1}{2}\rho U^2 \int_{-1}^{1} c_p \frac{dy}{d\xi}\, d\xi$$

$$= \tfrac{1}{2}\rho U^2 \int_{-1}^{1} \left(1 - \frac{1+\xi}{1-\xi}\right)\left(\frac{1-\xi}{1+\xi}\right)^{1/2} d\xi.$$

Normalizing the drag by the dynamic pressure $q = \tfrac{1}{2}\rho U^2$,

$$\frac{D}{q} = \int_{-1}^{1}\left[\left(\frac{1-\xi}{1+\xi}\right)^{1/2} - \left(\frac{1+\xi}{1-\xi}\right)^{1/2}\right] d\xi = \int_{-1}^{1}\left[\frac{-2\xi}{\sqrt{1-\xi^2}}\right] d\xi = 0.$$

We see that the forebody drag force is zero. How can this be? High pressure exists in the stagnation region near the re-entrant corner, and infinitely negative pressure exists at the convex corner, where the fluid velocity is infinite.

The line $y = 0$ represents a plane of symmetry for the problem of flow past a bluff forebody. Figure 3 is a sketch of the pressure distribution over the front surface of a bluff body in frictionless flow. The low pressure at the convex corners is called "leading edge suction." The forces from these two pressures cancel each other; the average pressure on the front face of the step is found to be exactly equal to the freestream pressure! Thus the drag on a semi-infinite body in potential flow is zero (see also references [3] and [4]). This fact has important consequences for the engineering design of vehicles that you see every day.

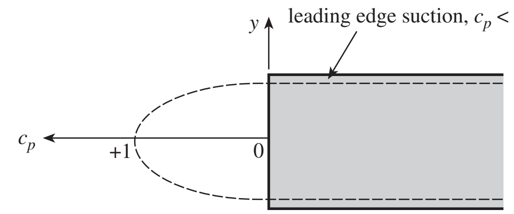

Figure 3 Ideal flow

If the fluid is not frictionless but instead has viscosity, a boundary layer forms next to any solid surface. The boundary layer may not always follow the contours of the body but may separate away from it in regions of rapidly rising pressure, which typically occur at the sharp corners. Thus the flow does not follow the sharp turn at the corner, and the low pressure there is lost. See Figure 4. However, if the surface contours are sufficiently gentle that the boundary layer can remain attached [5], then the flow outside the thin boundary layer is nearly that of a perfect fluid, with a corresponding pressure distribution. The leading edge suction is preserved, and the drag is nearly zero.

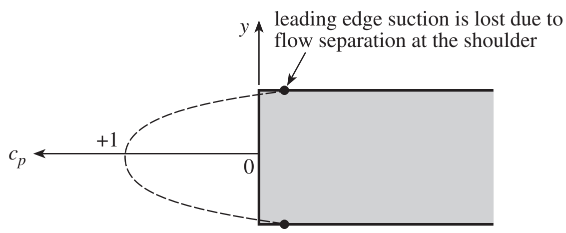

Figure 4 Real flow

Modern trucks (and cars) have a rounded shape to the front corners, so that the boundary layer can remain attached there, preserving the leading edge suction and dramatically reducing the drag. Without the rounding, the boundary layer would separate, the outer flow would see a much different effective shape, and the leading edge suction would be lost, corresponding to a much higher drag.

Airfoils have a rounded leading edge for the same reason. By maintaining attached flow there, the intense low pressure on the leading edge helps yield a very low drag. Turbine engine inlets likewise have a rounded inlet lip, so that at the start of a takeoff, the leading edge suction on the lip helps the aircraft accelerate. For a Boeing 747 at the start of the takeoff roll, about 40% of the entire thrust is due to leading edge suction pulling the aircraft forward! If you look closely, you can see the multiple rows of rivets securing the inlet lip to the rest of the engine nacelle.

Modern automobile and truck design exploits the leading edge suction to nearly eliminate the aerodynamic drag from their forebodies [1]. However, the drag from the base region on the backside of such bluff bodies is not so easily eliminated. Research continues on the aerodynamics of bluff bodies, making this subject much more interesting and complicated than that of streamlined objects [6].

Suggested Problems

1. Plot $c_p(y)$ for ideal flow past a two-dimensional step.
2. Estimate the drag if all leading edge suction is lost, such as from a sharp corner.
3. Estimate the energy saved per mile by one truck driving on level ground at 70 mph at sea level if it can preserve its leading edge suction by rounding off the upstream corners.

References

[1] *Aerodynamic Drag Mechanisms of Bluff Bodies and Road Vehicles* (ed. G. Sovran, T. Morel, & W. T. Mason). New York: Plenum, 1978.

[2] Hoerner, S. F., *Fluid Dynamic Drag.* Bricktown, N.J.: Hoerner Fluid Dynamics, 1965.

[3] Prandtl, L. and O. G. Tietjens, *Applied Hydro- and Aeromechanics* (translated by J. P. Hartog). London: Dover, 1934.

[4] Morel, T., “Theoretical lower limits of forebody drag.” *Aero. J.* 83 (1979): 23–27.

[5] Polhamus, E. C., “Effect of nose shape on aerodynamic characteristics of a body of revolution having a fineness ratio of 10.94.” *NACA RM L57F25* (1957).

[6] Koenig, K. and A. Roshko, “An experimental study of geometrical effects on the drag and flow field of two bluff bodies separated by a gap.” *J. Fluid Mech.* 156 (1985): 167–204.

Robert E. Breidenthal, Jr., is currently a professor in the Department of Aeronautics and Astronautics, University of Washington, Seattle. After graduating from Wichita State University, Mr. Breidenthal obtained both his master's and doctoral degrees from the California Institute of Technology. He is the author of several publications in his field of expertise.