Application Essay

A Matrix Model for Environmental Life Cycle Assessment

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Life cycle assessment (LCA) is an important tool for designing with environmental impact in mind, or designing for environment. Selecting product designs, materials, processes, reuse or recycle strategies, and options for final disposal should rely on a careful examination of the energy, resources, and environmental discharges associated with each alternative. Process and product models are commonly used for LCA in which materials and products are analyzed through different stages of fabrication, use, or end-of-life (reuse, recycling, disposal) options. LCA models have been developed as flow sheets for individual processes or as matrices based on economic input–output (EIO) data. In this example, we will demonstrate the use of matrix approaches to LCA.

In analyzing the environmental effects of products, note that products typically require more than one material input, so that multiple material process chains are required to capture all environmental impacts. For example, an LCA model for concrete includes the resource inputs of aggregates mining, cement production, chemical additives manufacturing, and electricity generation, and the environmental effects, or outputs, such as toxic emissions or hazardous wastes.

Consider a case in which reinforced concrete used to pave a 1-kilometer-long roadway costs \$150,000 and requires 3,680 metric tons of concrete (costing \$104,000) and 78 metric tons of reinforcing steel (costing \$46,000). This reinforced concrete product LCA is illustrated in a tabular, or matrix, form, shown in Table 1. For reinforced concrete, both iron and steel production (for reinforcing bars) and concrete are required. A column in this matrix represents the required direct inputs (from the rows) for a unit of output. For example, \$1,000 of concrete production requires \$120 of aggregate and \$200 of cement production, and \$1,000 of iron and steel production requires \$40 of iron and ferroalloy ores mining.

$$\mathbf{X}_{\text{direct suppliers}} = (\mathbf{I} + \mathbf{D})\mathbf{F}, \tag{1}$$

The direct supplier inputs for concrete production can be obtained by multiplying the matrix in Table 1 by the desired output of reinforced concrete:
where $\mathbf{X}_{\text{direct suppliers}}$ are the direct supplier inputs to reinforced concrete (in dollars), $\mathbf{I}$ is an identity matrix (to include the output of the concrete production stage

Table 1 A Process-Model-Requirements Matrix for \$1 of Reinforced Concrete Product

Inputs	Transportation	Aggregates Mining	Iron and Ferroalloy Ores Mining	Lime	Electricity	Coal Mining	Chemical Additives	Cement Production	Iron and Steel Production	Concrete Production
Transportation	0	0	0	0	0	0	0	0	0.03	0.10
Aggregates mining	0	0	0	0	0	0	0	0	0	0.12
Iron and ferroalloy ores mining	0	0	0	0	0	0	0	0	0.04	0
Lime	0	0	0	0	0	0	0	0	0.005	0
Electricity	0	0	0	0	0	0	0	0	0.07	0.04
Coal mining	0	0	0	0	0	0	0	0	0.05	0.02
Chemical additives	0	0	0	0	0	0	0	0	0.05	0.03
Cement production	0	0	0	0	0	0	0	0	0	0.20
Iron and steel production	0	0	0	0	0	0	0	0	0	0
Concrete production	0	0	0	0	0	0	0	0	0	0

Note. Rows represent inputs into the sectors named at column heads.

itself), **D** is the requirements matrix (shown in Table 1), and **F** is a vector of desired output. For $150,000 of reinforced concrete roadway, **F** consists of $104,000 of concrete and $46,000 of reinforcing steel.

For the most complete environmental assessment, LCA should take into account the entire supply chain for a product, including suppliers to a supplier, called indirect suppliers. Consider, for example, that iron and steel production contributes to aggregates mining, iron and ferroalloy ores mining, cement production, concrete production, even to iron and steel production. Therefore, each sector of the economy named in the column heads has inputs represented by rows. In Table 1, we only have nonzero values for the direct suppliers to the concrete and reinforcing steel processes, so the supply chain is only one level deep. The second level supplier requirements could be calculated as **D*D*F,** but in some cases third or fourth levels of suppliers exist also. The total output for the various stages of LCA can be calculated as the series:

$$\mathbf{X} = (\mathbf{I} + \mathbf{D} + \mathbf{D{*}D} + \mathbf{D{*}D{*}D} + \ldots)\mathbf{F}, \qquad (2)$$

where **X** is the change in total output (in dollars), **I** is an identity matrix (to include the output of the concrete production stage itself), **D** is the requirements matrix (shown in Table 1), and **F** is a vector representing the desired final demand (of concrete in this case). The supplier requirements series ($\mathbf{I} + \mathbf{D} + \mathbf{D{*}D} + \mathbf{D{*}D{*}D} + \ldots$) can be approximated by $(\mathbf{I} - \mathbf{D})^{-1}$, so the total output including indirect suppliers is:

$$\mathbf{X} = (\mathbf{I} - \mathbf{D})^{-1}\,\mathbf{F}. \qquad (3)$$

The $(\mathbf{I} - \mathbf{D})^{-1}$ table is illustrated in Table 2.

Once the economic output for each stage is calculated, then a vector of direct environmental outputs can be obtained by multiplying the output at each stage by the environmental impact per dollar of output:

$$\mathbf{B}_i = \mathbf{R}_i\mathbf{X} = \mathbf{R}_i(\mathbf{I} - \mathbf{D})^{-1}\mathbf{F} \qquad (4)$$

where $\mathbf{B}_i$ is the vector of environmental burdens (such as toxic emissions or electricity use), and $\mathbf{R}_i$ is a matrix with diagonal elements representing the impact per dollar of output for each stage. Illustrated below is an **R** matrix in which the elements are given in kilograms of Resource Conservation and Recovery Act (RCRA) Subtitle C hazardous waste generated per $1,000 of output. For example, 0.027 kg of hazardous waste are generated per $1,000 of output from the coal-mining sector.

Table 2 Ten-sector Total Requirements for $1 of Each of the Inputs to the Example Reinforced Concrete Product

Inputs	Transportation	Aggregates Mining	Iron and Ferroalloy Ores Mining	Lime	Electricity	Coal Mining	Chemical Additives	Cement Production	Iron and Steel Production	Concrete Production
Transportation	1.19700	0.02556	0.02408	0.05055	0.00984	0.01426	0.02865	0.05523	0.03322	0.10076
Aggregates mining	0.00010	1.04037	0.00062	0.00124	0.00059	0.00178	0.00022	0.00168	0.00070	0.12402
Iron and ferroalloy ores mining	0.00013	0.00046	1.00745	0.00035	0.00021	0.00052	0.00139	0.00215	0.04099	0.00063
Lime	0.00005	0.00008	0.00056	1.00332	0.00013	0.00032	0.00082	0.00017	0.00488	0.00009
Electricity	0.01683	0.05801	0.20138	0.08399	1.00861	0.03367	0.05604	0.12515	0.07199	0.04477
Coal mining	0.00212	0.00889	0.02507	0.07788	0.11145	1.12447	0.01201	0.10313	0.04676	0.02265
Chemical additives	0.00583	0.00892	0.02152	0.01837	0.00582	0.00830	1.25858	0.01621	0.04903	0.03182
Cement production	0.00026	0.01429	0.00052	0.00208	0.00074	0.00046	0.00059	1.03467	0.00049	0.19756
Iron and steel production	0.00323	0.00963	0.01261	0.00540	0.00491	0.01154	0.00936	0.00601	1.16573	0.00464
Concrete production	0.00029	0.00122	0.00071	0.00077	0.00160	0.00057	0.00047	0.00091	0.00092	1.01104

Note. Rows represent inputs into the sectors named at column heads.

$$\begin{pmatrix} 2.096 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.002 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.162 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.025 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.745 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.027 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 94.158 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13.866 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 42.454 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0002 \end{pmatrix}$$

EXERCISES

1. Calculate the direct supplier inputs to reinforced concrete ($\mathbf{X}_{\text{direct suppliers}}$) in thousands of dollars, corresponding to the example presented above.

Solution:

$\mathbf{X}_{\text{direct suppliers}} = (\mathbf{I} + \mathbf{D})\mathbf{F} =$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.03 & 0.10 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.12 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0.04 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0.005 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0.07 & 0.04 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0.05 & 0.02 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0.05 & 0.03 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0.20 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 46 \\ 104 \end{pmatrix} = \begin{pmatrix} 12 \\ 12 \\ 2 \\ 0.2 \\ 7 \\ 4 \\ 5 \\ 21 \\ 46 \\ 104 \end{pmatrix}$$

2. Interpret the results from Exercise 1. How much aggregate mining is required for \$150,000 of concrete?

Solution: \$12,000

3. How much RCRA hazardous waste is generated for the direct suppliers in the example presented above, in kilograms, using (4)? (Use $\mathbf{X}_{\text{direct suppliers}}$ from Exercise 1.)

Solution:

$\mathbf{B} = \mathbf{R}\,\mathbf{X}_{\text{direct suppliers}} =$

$$\begin{pmatrix} 2.096 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.002 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.162 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.025 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.745 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.027 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 94.158 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13.866 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 42.454 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0002 \end{pmatrix} \begin{pmatrix} 12 \\ 12 \\ 2 \\ 0.2 \\ 7 \\ 4 \\ 5 \\ 21 \\ 46 \\ 104 \end{pmatrix} = \begin{pmatrix} 25 \\ 0.03 \\ 0.3 \\ 0.005 \\ 19 \\ 0.1 \\ 471 \\ 291 \\ 1{,}953 \\ 0.02 \end{pmatrix}$$

4. Interpret the results from Exercise 3. How much RCRA Subtitle C hazardous waste is generated for \$4,000 of coal mining?

Solution: 0.027 kg of hazardous waste is generated per \$1,000 of output in the coal-mining sector, thus for \$4,000 of coal mining, 0.1 kg of hazardous waste is generated.

5. Overall, how much RCRA hazardous waste is generated for the direct suppliers, as calculated in Exercise 3?

Solution: 2,759 kg of RCRA Subtitle C hazardous waste are generated (obtained by summing the elements of the **B** vector).

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Reference

[1] Hendrickson, C. T., A. Horvath, S. Joshi, and L. B. Lave, "Economic Input–Output Models for Environmental Life Cycle Assessment." *Environmental Science and Technology, ACS,* 32, no. 4 (1998): 184A–191A.

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